THERMODYNAMIC ASPECT OF SHORT-TERM FREQUENCY STABILITY OF DIRECTLY HEATED RESONATORS

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Abstract.

This paper is an attempt to study a problem of degradation of short-term stability (STS) of OCXO based on the directly heated resonators (DHR) due to variations in the heating power. The analytical model linking the DHR frequency fluctuations with the heating current and ambient temperature variations, as well as the DHR properties has been obtained and verified experimentally. The carried out research allowed development of the means for designing DHR with STS better than 5E-12 per 1 s, which is comparable to the best results attainable with the conventional technique of heating the crystal.

1. Introduction.

The directly heated resonator (DHR) technology provides significant reduction in warm-up time, size, and power consumption of OCXO thanks to employing thin film heater and thermistor directly deposited on the crystal surface [1]. Frequency stability of such devices, however, is essentially dependent on the temperature gradients over the crystal plate. The gradients are produced by power dissipated in the film heaters. Variations in the ambient temperature and/or supply voltage cause variations in the power dissipated by the heaters, which in turn leads to thermal gradients variation. This effect results in degradation of temperature and/or supply voltage frequency sensitivity of the OCXO. Periodic or random variations of the heating power caused by changes in external conditions or noise in thermocontroller circuitry translate into fluctuations of the DHR frequency through thermodynamic sensitivity of the crystal plate. That leads to degradation of STS and close-to-the-carrier phase noise performance of the OCXO employing such resonators.

A goal of the present paper is theoretical and experimental analyses of the influence of the thermodynamic effects on STS of the DHR, which would ultimately lead to the development of effective means of their prediction and improvement.

1. Model of the heating power fluctuations.

To understand an origin of the heating current fluctuation in the DHR let's consider its construction along with the thermocontroller circuit used to accurately control the crystal temperature. The thermocontroller circuit is shown schematically in Fig. 1. It consists of a thermo-sensitive bridge, an amplifier and a regulating transistor governing the current through the heater resistor deposited on the crystal plate. The DHR construction (Fig.2) contains inside the vacuum holder the crystal plate with the film heaters and the thermistor arranged on its surfaces. The thermocontroller circuitry can be partly or entirely located inside the resonator volume [2]. Effective thermal insulation of the heated part of the DHR from environment is used to minimize the heating power consumption.

For the described circuit the power through the film heaters in the steady state can be expressed as:

$$P_s = E_c * I_s * K_a * \Delta R_t * / 4R1, \quad (1)$$

where E_c - voltage on the thermo-sensitive bridge; K_a - the amplification coefficient of the circuit; ΔR_t – the bridge misbalance providing the steady heating current I_s,
R1 – resistance of the bridge resistors.

The power dependence on the thermocontroller parameters and ambient temperature deviation can be found as a derivation of (1):

$$dP_s=I_s^2 h\, dE_c/E_c + I_s K_a E_c\, \alpha\, dT_c(4R1),\ (2)$$

where α- temperature coefficient of the thermistor resistance; dT_c – temperature variation of the thermistor; $\alpha\, dT_c=d\Delta R_t$.

As one can see from the expressions the power variations are dependent on the thermistor temperature and on the input voltage changes. The power variations decrease with reduction of steady state heating current and of the amplification coefficient. The input voltage changes are translated directly into the power fluctuations. So effective voltage regulator should be used with the thermo-sensitive bridge to eliminate influence of the voltage supplier on the DHR frequency.

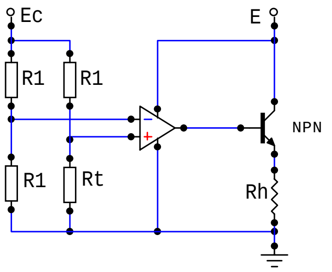

Fig.1.

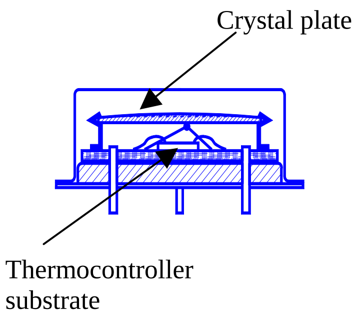

Fig. 2.

The temperature fluctuations in the thermistor result from environmental temperature fluctuations. Electrical network simulating the temperature variations of the crystal versus the ambient temperature deviations is shown in Fig. 3.

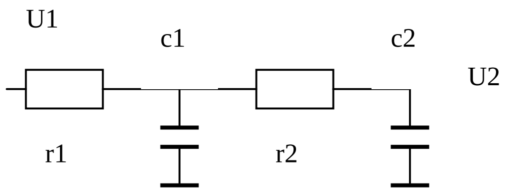

Fig.3.

In this model r1 – is an electrical equivalent of the thermal resistance between the holder and the environment, r2 – is an equivalent of the thermal resistance between the crystal and the holder, c1, c2 – are electrical equivalents of the thermal capacity of the holder and the crystal plate respectively; U1 – simulates ambient temperature variation $dT_a(\Omega)$ with frequency Ω; U2- simulates the temperature variation of the crystal $dT_c(\Omega)$.

From the model one can find the crystal temperature variation $dT_c(\Omega)$ versus amplitude and frequency of ambient temperature $dT_a(\Omega)$ changes:

$$dT_c(\Omega)=dT_a(\Omega)/\sqrt{(1+\Omega^2\tau 1\tau 2)^2+\Omega^2(\tau 1+\tau 2+\tau 12)^2}$$

where $\tau 1$ = c1* r1; $\tau 2$ = c2*r2; $\tau 12$ = c1*r2. Since $\tau 1$, $\tau 2$, $\tau 12 >> 1$, the expression can be simplified:

$$dT_c(\Omega) \cong dT_a(\Omega)/\Omega^2\tau 1\tau 2,\quad (3)$$

Hence from expression (3) the crystal temperature fluctuations decline with the improvement of the effectiveness of the thermal insulation and with the increase of frequency of the temperature variations. For a DHR packaged in the TO-8 vacuum holder with the thermal insulation resistance r2=500 K/W the ambient temperature variations at 1 Hz are translated into the crystal's temperature being reduced to 1E-5 of dT_a while 0.01 Hz temperature variations pass into the crystal plate being reduced to 0.1 of dT_a.. Improvement of the insulation leads to proportional reduction of the crystal temperature fluctuations.

Substitution of expression (3) in (2) allows determination of the heating power fluctuations versus ambient temperature variations, the thermocontroller parameters and the DHR thermal properties. Next stage of the work determines frequency changes induced by the power fluctuations in the crystal plate.

3.General equation for the crystal frequency changes induced by a stress field.

It's known that temperature gradients in the crystal plate produce non-uniform mechanical stress field resulting in frequency shift via variation of the stiffness coefficients. Due to non-uniform distribution of the vibration amplitude over the plate the resulting frequency deviation of the crystal is a function of both the fields interaction. To define the frequency shift of the crystal under the stress field let's consider the energy balance equation of the plate vibrating in the thickness-shear mode:

$$K_{max} = P_{max}, \qquad (4)$$

K_{max} is maximal kinetic energy while the vibration period:

$$K_{max} = \omega\, \rho \int u^2 (x,z)\sin^2(m\pi y/2h)ds$$

and the maximal potential energy:

$$P_{max}=(m^2/4h^2) \int q\, u^2 (x,z) \cos^2(m\pi y/2h)dv$$

Here u (x,z) – is a distribution of the thickness-shear mode displacements over the crystal plate; ρ - density of quartz, m - overtone number of the vibrating mode, q – stiffness coefficient of the vibrating mode under the stresses; 2h – thickness of the plate.

Substituting expressions for K_{max}, P_{max} in (3) and taking that $q = q_o + dq(x,z)$, and $\omega= \omega_o +\Delta\omega$ (where q_o – the stiffness coefficient of the crystal mode in absence of the stress field; $dq(x,z)$ – variation of the crystal stiffness under the stress field $\sigma(x,z)$, ω_o- resonance frequency of the crystal in the absence of the stress field, $\Delta\omega$ - the frequency change due to the stress field, we get the balance equation for the crystal vibrating under the stress field:

$$\rho(\omega_o + \Delta\omega)^2\int A^2(x,z)\, ds = (m/2h)^2 \int (q_o + dq(x,z))\, A^2 (x,z)ds, \qquad (4)$$

where A(x,z) – normalized vibrations amplitude over the plate equaled to:

$$A(x,z)=\exp(-x^2/a^2-z^2/b^2),$$

where a, b – active area sizes along x and z axis of the crystal plate.

Since $(\omega_o+\Delta\omega)^2 \approx \omega_o^2 + 2\Delta\omega\omega_o$, and $\omega_o = (\pi m/2h)^2(q_o/\rho)$, expression (4) can be written as:

$$\rho(\omega_o^2+2\Delta\omega\omega_o)\int A^2(x,z)ds = (\pi m/2h)^2\int dq(x,z)A^2(x,z)ds + \omega_o\int A^2(x,z)ds$$

Solving the equation for the frequency shift $\Delta\omega$, we obtain:

$$\Delta\omega =(\pi\, m/2h\omega_o)^2 \int dq(x,z)\, A^2(x,z)\, ds\, /\, (2\rho \int A^2(x,z)\, ds$$

Dividing the equation by ω_o we come to the expression for fractional frequency changes:

$$\Delta\omega/\omega_o = \int dq(x,z)\, A^2(x,z)ds\, /\, (2q_o \int A^2(x,z)\, ds)(5)$$

The crystal plate can be imagined as a composition of small areas ds with coordinates x, z being under the uniform stresses of $\sigma(x,z)$ value (within ds) which produce the frequency change $\partial\omega/\omega_o$ due to the stiffness coefficient variation:

$\partial\omega/\omega_o = \partial q/2q_o = K_\sigma(x,z)\ \boldsymbol{\sigma}(x,z)$,

where $K_\sigma(x,z)$ - coefficient equaled to the frequency change produced by the uniform stresses of $\boldsymbol{\sigma}(x, z)$ along axis x or z. Then equation (5) can be written as:

$$\Delta\omega/\omega = \int K_\sigma(x,z)\ \sigma(x,z)\ A^2(x,z)\ ds \,/ \int A^2(x,z)\ ds \quad (6)$$

Obtained equation allows calculation of the crystal frequency changes produced by arbitrary field of mechanical stresses $\sigma(x,z)$ and in fact is similar to Tiersten's perturbation integral.

In case of the circular crystal plate under the symmetrical (about the plate center) stresses field equation (6) can be written as

$$\Delta\omega/\omega = K_\sigma \int \sigma(r)\ A^2(r)\ dr \,/ \int A^2(r)\ dr, \quad (7)$$

where K_σ – integral frequency-stress coefficient dependent on the crystal cut only. For the case of non-uniformly heated plate the thermal stress field $\sigma(r)$ is a linear function Ψ of the thermal gradient field $\partial T/\partial r$. Thus, equation (6) can be written as:

$$\Delta\omega/\omega = \int K_\sigma(x,z)\ \Psi(x, z)\ A^2(x, z)\ ds \,/ \int A^2(x,z)\ ds \quad (8)$$

For the similar boundary conditions and the same plate geometry following equation is valid:

$$\Psi_1(x,z)/\Psi_2(x,z) = (\partial T_1/\partial x)/(\partial T_2/\partial x), \quad (9)$$

where $\partial T_1/\partial x$, $\partial T_2/\partial x$ – thermal gradients produced by some temperature fields $T_1(x)$ and $T_2(x)$.

4. Calculation of the thermal gradients in the plate and their influence on the crystal frequency.

Typical designs of the DHR crystal plate are shown in fig. 4. The crystal plates can be circular or rectangular with the film heaters deposited on the peripheral. In the rectangular plates the thermal power flows from the heaters into the central part of the plate along one axis only. So, a one-dimension model can be used for definition of the thermal gradient pattern. The thermal gradient in the circular plate with sufficient accuracy can be estimated as a superposition of two one-dimension solutions in the square plate with thermal flows along **x** and **z** axes (fig. 4 c).

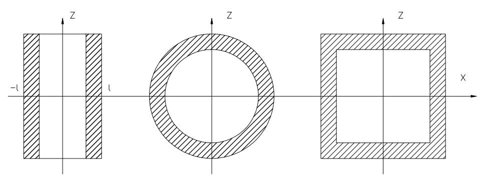

Fig.4.

At ambient temperature noise the power variation in the heater is a sum of periodical functions:

$$\Delta\mathbf{P} = \sum \Delta P_{oi} \cos(\Omega_i t),$$

where ΔP_{oi}, Ω_i–amplitude and frequency of i-component of the power variations.

For one-dimensional model the temperature field is described by the differential equations:

$$\partial\mathbf{T}/\partial x = R\ \Delta\mathbf{P},$$
$$-\partial\Delta\mathbf{P}/\partial x = j\Omega C\mathbf{T} \quad (10)$$

at the boundary conditions:

$$\Delta\mathbf{P} = ½\ \Delta P_{oi} \cos(\Omega_i t) \ \text{ at } \ x = l,$$
$$\Delta\mathbf{P} = -½\ \Delta P_{oi} \cos(\Omega_i t) \ \text{ at } \ x = -l, \quad (11)$$

where **T**, $\Delta\mathbf{P}$ – complex temperature and the power variations respectively; R – thermal resistance of the plate equaled to $1/(hl\lambda_x)$ (λ_x-thermal conductivity of quartz crystal along **x**-axis); $C = c_o\rho hl$ (c_o - thermal capacity of the plate).

Solution of system (10), (11) about the temperature gradient ($\partial \mathbf{T}/\partial x$) can be written as:

$$\partial T/\partial x = R\sum \Delta P_{oi} \cos(\Omega_i t)|sh(z1)/h(z2)|, \quad (12)$$

where

$z1 = \alpha(l-x) + j\beta(l-x)$,

$z2 = \alpha l + j\beta l$

and

$\alpha = \beta = \sqrt{\Omega\, c\rho/2\lambda}$

Calculation of the temperature gradients over a SC-cut plate with 10 mm width and 0.55 mm thickness in dependence on the fluctuation frequency Ω is shown in Fig. 5. As one can see the temperature gradients reach maximum at the plate edges and fall to zero in the plate center. The gradients decrease with raise of frequency Ω: for low Ω<0.1 Hz the gradients are linearly distributed over the plate while for higher Ω the gradients decrease essentially in the plate center region.

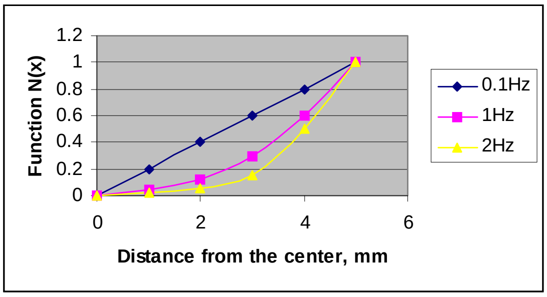

Fig. 5.

Value of the gradients depends on the crystal thermal conductivity, so the overtone rectangular plates with the heaters deposited along crystallographic X' axes are less sensitive to the power variations. Taking into account dependence of the crystal temperature fluctuations on the ambient temperature (see exp. 4) one can conclude that only very slow temperature variations (at $\Omega < 0.01$ Hz) can produce noticeable gradients in the crystal plates.

Functions $\Psi(\partial T/\partial x)$ in (8) is the most difficult for determination as being dependent on a variety of the DHR parameters, i.e. the crystal plate cut and geometry, the heater configuration, as well as of the manufacturing imperfections. That makes theoretical definition of the function hardly practical.

To avoid calculation of Ψ we used experimental method of determination of thermodynamic sensitivity of the crystal plates based on measurement of "overshoot" of their frequency during warming-up. The "overshoot" phenomena is well studied at comparatively slow the heating speeds - below than 1°C/s. We studied fast heating process at about 10°C/s for SC-cut and modified SC-cut (φ=23°25'; θ=34°) crystals (Fig. 6).

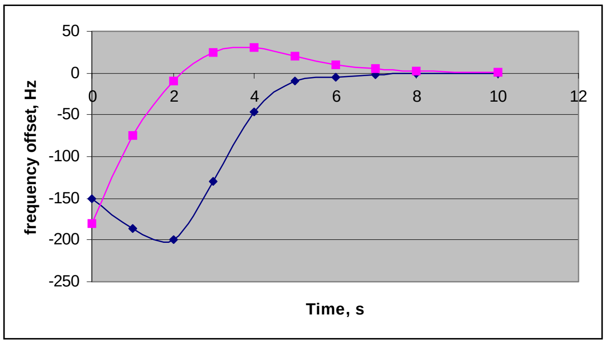

Fig. 6

As it's seen form the curves sign and amplitude of the "overshoot" are different for these crystals. The "pure" SC-cut crystal "overshoot" is negative with 4-6 ppm amplitude while for the modified SC-cut it has positive sign with maximum of about 3 ppm. Obviously, "zero-overshoot" cut must be oriented at about 23°00'.

Analysis of the warming-up process reveals that the "overshoot" is caused by thermal gradients between the plate center and its edges (where the thermistor is located) appeared by the moment when temperature of the thermistor reaches a preset value.

Process of heating the crystal plate at the start-up is governed by the differential equation of thermal conductivity:

$$\partial T^2/\partial x^2 = (\rho c_o/\lambda)\, \partial T/\partial t \qquad (13)$$

at the boundary conditions:

$$\text{at } x = -l \quad \partial T/\partial x = (\rho c_o/\lambda)\, P_o/2,$$
$$\text{at } x = l \quad \partial T/\partial x = -(\rho c_o/\lambda)\, P_o/2, \qquad (14)$$

where P_o – start power in the heaters.

Solution of equation (13) for time $t>\rho c_o l^2/\lambda$ (about 2 s for the 3rd overtone 10 MHz crystals) allows determination of the gradient values resulting in the frequency "overshoot":

$$(\partial \mathbf{T}/\partial x)_{ov} = P_o\, (x/2\lambda h l^2)$$

Then frequency "overshoot" can be found from (11):

$$(\Delta\omega/\omega)_{ov} = \int K\sigma(x,z)\, \Psi(x, z)_{ov}\, A^2(x, z)\, ds / \int A^2(x,z)\, ds \qquad (15)$$

On the other hand, frequency deviation due to heating power fluctuations can be expressed as:

$$(\Delta\omega/\omega)_p = \int K\sigma(x,z)\, \Psi(x, z)_p\, A^2(x, z)\, ds / \int A^2(x,z)\, ds, \qquad (16)$$

where $\Psi(x, z)_p$ is the function of thermal gradients $(\partial T/\partial x)_p$ produced by power fluctuations in the heaters and governed by equation (12).

Taking into account (9) from equations (15), (16) one can determine frequency deviations caused by the heating power variations:

$$(\Delta\omega/\omega)_p = (\Delta\omega/\omega)_{ov} \int (\partial T/\partial x)_p\, A^2(x,z)\, ds / \int (\partial T/\partial x)_{ov}\, A^2(x,z)\, ds \qquad (17)$$

Substituting expressions for $(\partial T/\partial r)_p$ and $(\partial T/\partial r)_{ov}$ in (17) we obtain equation for frequency fluctuations due to the heating power variations:

$$(\Delta\omega/\omega)_p=(\Delta\omega/\omega)_{ov}(\Delta P_s/P_o) \int l\, N(x) A^2(x,z) ds / \int x\, A^2 (x,z)\, ds, \qquad (18)$$

where $N(x)= |sh\,(z1)/sh\,(z2)|$.

Function $N(x)$ at small x (near a center of the plate) can be approximated by a linear function: $N(x)=N(a)(x/a)$, where a – is a size of an active area of the plate. Integrating (17) we obtain simple expression for the frequency fluctuations versus the heating power variations:

$$(\Delta\omega/\omega)_p= (\Delta\omega/\omega)_{ov}\, (\Delta P_s/P_o)(l/a)N(a) \qquad (19)$$

Substituting equations (2), (3) into (18) one can calculate the frequency fluctuations versus ambient temperature variations:

$$(\Delta\varpi/\varpi)_t = (\Delta\varpi/\varpi)_{ov}\, N(a)\, (l/a)\, I_s\, K_a\, E_c\, \alpha\, \Delta T_a \mathbf{(\Omega)/(\Omega^2}\, \tau_1\, \tau_2\, 4R_1\, P_o) \qquad (20)$$

The crystal frequency deviation dependence on the temperature fluctuations frequency was calculated (Table 1) for 10 MHz 3d overtone SC-cut DHR with the following properties: thermal insulation resistance 500 K/W; frequency "overshoot" $(\Delta\varpi/\varpi)_{ov}$ = 3 ppm; τ_1 = 70 s; K_a = 20; E_c = 8V; α=1KOhms/°C and ΔT_a = 0.1°C).

Table 1

Temperature fluctuation frequency, Ω, Hz	Crystal Frequency Deviation, $\Delta f/f$
0.01	1E-11
0.1	1E-13
1.0	1E-15

Fig.7

As it follow from the data noticeable (above 1E-11) DHR frequency deviations take place only at slow ambient temperature variations – below 0.01 Hz. However tests of OCXOs based on different types of DHR show noticeable variation of their heating current (within 0.005 - 0.01 mA) resulting in considerable degradation of the STS. The most likely reason for the current fluctuations is noise in the thermocontroller circuitry components, primarily in the thermistor.

To calculate dependence of the DHR frequency on the heating current fluctuation expression (19) can be transformed by substituting $\Delta P_{si} = 2I_{si} R_h\, dI_s$ and $P_o = I_o^2 R_h$:

$$(\Delta f/f)_i = 2(\Delta f/f)_{ov} (l/a) N(a)\, dI_s I_s/I_o^2 \quad (21)$$

Equation (21) allows prediction of the DHR frequency fluctuations on the basis of the heating current fluctuation measurements regardless of its origin. It was applied for calculation of STS of different types of DHR. The results are shown in Table 2 in comparison with experimental data obtained for the OCXO built using different types of DHR.

Table 2.

Type of the DHR design	Steady state power in the heaters, mW	Start–up current/steady state current, mA	Overshoot, ppm	Current variations at 1Hz, mA	Frequency fluctuations, E-11	
					theory	test
A	60	30/12	3.0	0.005	20	15
B	3.5	70/5	1.0	0.005	0.6	0.8
C	6.0	200/11	3.0	0.01	0.9	1.0

A. Glass packaged DHR, AT-cut, 5th overtone, 10 MHz
B. Glass packaged DHR, modified SC-cut, 3rd overtone, 10 MHz
C. TO-8 packaged RT, modified SC-cut, 3rd overtone, 10 MHz

As one can see the theoretical results are close to the experimental data. For the OCXO based on the AT-cut DHR design the STS is 20-30 times worse as compared with that of the SC-cut DHR that results from significant thermodynamic sensitivity of the AT-cut crystals and considerable power dissipated in the heaters.

On the basis of above consideration following means of reduction in the DHR frequency fluctuations caused by the ambient temperature deviations and inherent noise in the thermocontroller can be proposed:

1. Reduction of power consumption of the DHR is the most effective way to improve STS. It effectively shields the crystal from transferring the ambient temperature variations. The result is proportional reduction in heating current fluctuation.
2. The translation of the crystal temperature deviation into the heating power fluctuations can be reduced by reduction of the thermocontroller amplification coefficient, the thermistor coefficient α and the thermo-sensitive bridge input voltage E_c. These methods however are limited by possible degradation of the DHR temperature stablity.
2. Thermo-dynamic sensitivity of the crystal plate should be minimal, which demands application of SC-cut plates with optimized configuration of the heaters. Moreover the 5th overtone crystals are preferred over the 3d overtone ones while the later have better STS in comparison with the fundamental mode crystals.
3. Low-noise components of the thermocontroller circuitry should be used to minimize its inherent noise resulting in fluctuations of the heating current.

As concluded above, the most effective way to improve DHR STS is a reduction of the heating power by improvement of the thermal insulation. We've developed ultra-low consumption SC-cut DHR packaged in TO-8 vacuum holder. Its thermal insulation resistance is about 1000 W/K which results in 5 mA steady state current and thermal power dissipated in the crystal below 1 mW. The STS of such designs is measured at about 5E-12 per 1s, the Single Side Band Phase Noise Power Density at 1 Hz offset is below –90 dBc/Hz.

References:

1. I.Abramzon, «OCXO Design Using Composite-Heating of the Crystal Resonator», IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, Vol.41, No.2, 1994, pp.284-287.
2. I.Abramzon, R. Boroditsky, D. Cocuzzi, «Miniature OCXOs Using DHR Technology», Proc. of 1997 IEEE AFCS, pp. 943-947.
3. Tiersten

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